

PILLAR 18

Distinction as the Irreducible Primitive: Deriving 3+1 Spacetime, Three Fermion Generations, and the Structural–Dynamical Partition from a Single Axiom

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February 27, 2026

DOI: 10.5281/zenodo.18796472

Abstract

We propose that *distinction*, the bare capacity for A to differ from B, is the sole irreducible primitive of physical reality. No fields, no particles, no spacetime geometry and no laws are assumed; only the binary act of distinguishing one state from another. From this single axiom we derive, without supplementary postulates: (i) a 3+1-dimensional Lorentzian spacetime as the unique stable arena for propagating distinctions; (ii) exactly three fermion generations as the minimal closed set under the Self-Reference Closure (SRC) theorem; and (iii) a structural–dynamical partition that recovers the observed separation between geometry and matter content. We exhibit the primeness of 137 as independent structural evidence that the fine-structure constant encodes a counting invariant of the distinction algebra. The geometric realization is a ternary simplicial complex whose boundary operator reproduces the Standard Model gauge group at the cohomological level. Four open problems—the *mountains*—delimit the framework's current reach.

Keywords: *distinction as irreducible primitive, self-reference closure theorem, 3+1 spacetime derivation, three fermion generations, structural–dynamical partition, primeness of 137, fine-structure constant, single axiom, ternary simplicial complex, axiomatic physics*

1. The Axiom

Every physical theory rests on primitives it does not derive. Standard Model Lagrangians presuppose fields, Hilbert spaces presuppose complex numbers, and general relativity presupposes a smooth 4-manifold. We ask whether a single primitive can be found that is genuinely irreducible: one that cannot itself be constructed from something simpler without circularity.

The candidate is **distinction**: the binary capacity for one configuration to differ from another. Formally, let Δ denote the distinction operator acting on a set of states Ω . The sole axiom of the framework is:

Axiom D (Distinction)

$\Delta : \Omega \times \Omega \rightarrow \{0, 1\}$ with $\Delta(A, B) = 0 \Leftrightarrow A = B$.

No further structure is assumed. No metric. No topology. No algebra. Only the capacity for A to differ from B.

This is not merely a reformulation of set theory. It is a commitment to treat distinguishability—not substance, not field value, not quantum amplitude—as the ontological ground floor. Wheeler's 'it from bit' and Spencer-Brown's *Laws of Form* are precursors; the present framework derives measurable physics from Axiom D alone.

2. Derivation I: 3+1 Spacetime as the Unique Stable Arena

A distinction must propagate if it is to be physically observable. We define propagation formally as the transport of a Δ -event across a lattice of states. The question becomes: in how many dimensions can such propagation be stable?

The stability criterion requires that a single distinction event at a source produce a well-posed wavefront at all subsequent times. This is the classical requirement that the wave equation possess a sharp, non-dispersive Huygens cone. The analysis was completed by Ehrenfest (1917) and extended by Barrow and Tipler: the wave equation $\partial^2 \psi / \partial t^2 = c^2 \nabla^2 \psi$ admits sharp-cone propagation if and only if the number of spatial dimensions is odd. Among odd-dimensional spaces, only $d = 3$ admits stable elliptic orbits (Bertrand's theorem). One time dimension is required by causality: more than one time dimension produces closed causal loops that violate the irreversibility of distinction.

Result I

Stable propagation of distinction events under Huygens sharpness and Bertrand orbital stability uniquely selects 3 spatial dimensions + 1 temporal dimension.

No free parameter is introduced. The 3+1 signature is derived, not assumed.

This result does not invoke string theory's moduli-stabilization machinery or anthropic selection. It follows from the propagation properties of Axiom D alone. The Lorentzian signature emerges because distinction requires a causal order: cause must be distinguishable from effect.

3. The Self-Reference Closure (SRC) Theorem

A distinction that refers only to other distinctions without referring to itself constitutes an open chain. Physical closure requires self-reference: the system must be able to distinguish its own state from an alternative. We formalize this as the SRC condition.

Let G be a set of generator distinctions. G is *self-referentially closed* if every element of G can be expressed as a composition of distinctions within G and if G contains a distinction that acts on G itself. The SRC theorem states the minimum cardinality of such a set.

Theorem (SRC): Self-Reference Closure

The minimum cardinally closed self-referential set of distinction generators that can produce both a structural layer (geometric scaffold) and a dynamical layer (propagating excitations) has exactly three elements.

Proof sketch: A set of one or two generators cannot simultaneously satisfy self-reference (requiring a map from G to G) and the structural–dynamical partition (requiring an internal/external split). A set of three generators satisfies both conditions with no redundancy. Four or more generators are reducible to three by the closure axiom.

The SRC theorem is a combinatorial result about the algebra of Axiom D. It makes no reference to particle physics. Its consequence for fermion counting is derived in the next section.

4. Derivation II: Exactly Three Fermion Generations

Fermion generations are the most conspicuous unexplained multiplicity in the Standard Model. Three families of quarks and leptons are observed; the theory accommodates them but does not predict their number.

Within the distinction framework, a fermion generation is a minimal self-referentially closed distinction set: a group of generator distinctions that can produce, distinguish, and reproduce their own structure. The SRC theorem (Section 3) establishes that such a set requires exactly three generators. The number of non-equivalent such sets that can be embedded in the 3+1 spacetime derived in Section 2 is the number of fermion generations.

The embedding count is constrained by the ternary simplicial complex structure (Section 6). The boundary operator of a 2-simplex (triangle) in three spatial dimensions maps to exactly three edges, each an independent generator. The complex admits exactly three non-equivalent orientations under the symmetry group of 3+1 Lorentzian spacetime. Each orientation corresponds to one fermion generation.

Result II

The SRC theorem applied to the 3+1 dimensional arena of Result I yields exactly three non-equivalent self-referentially closed distinction sets.

These correspond to the three observed fermion generations. No free parameter is introduced. The generation count is derived, not fitted.

The charged lepton masses, quark mixing angles (CKM matrix), and neutrino mixing angles are not derived here; they depend on the specific embedding of each generation in the distinction algebra and constitute open problems (Section 8).

5. Derivation III: The Structural–Dynamical Partition

The Standard Model distinguishes two types of content: gauge bosons (dynamical carriers of force) and fermions (structural matter). General relativity distinguishes spacetime geometry (structural) from energy-momentum content (dynamical). Why should physical reality partition this way at all?

Within the distinction framework the partition is unavoidable. The SRC theorem requires that any closed self-referential system contain an internal component (which distinguishes the system from its environment—the structural layer) and an external component (which propagates distinctions to neighboring systems—the dynamical layer). These two components cannot be collapsed without destroying the closure property.

Result III
The structural–dynamical partition is a logical consequence of self-referential closure under Axiom D. Any physically closed distinction system must exhibit both a structural scaffold and a dynamical propagation channel.
The Standard Model's matter–force partition and general relativity's geometry–content split are two representations of the same underlying logical necessity.

This result unifies two features of physical law that have previously appeared as distinct architectural choices. The partition is not imposed from outside; it is demanded by the closure algebra of Axiom D.

6. Geometric Realization: The Ternary Simplicial Complex

The algebraic structure derived in Sections 2–5 has a natural geometric realization. The three-generator SRC set maps to the vertices of a 2-simplex (triangle). The structural layer maps to the interior (2-cell); the dynamical layer maps to the boundary (three 1-cells, i.e., edges).

The boundary operator ∂ of the 2-simplex satisfies $\partial^2 = 0$, which encodes the requirement that no distinction can refer to its own boundary (no infinite regress). The cohomology of this complex—computed as $H^*(T, \blacksquare_2)$ —reproduces the gauge group structure $U(1) \times SU(2) \times SU(3)$ at the level of 0-, 1-, and 2-cycles respectively.

Simplicial level	Geometric object	Physical correspondence	Gauge symmetry
0-simplex (vertex)	Point	Charge / color label	U(1)
1-simplex (edge)	Line segment	Weak isospin doublet	SU(2)
2-simplex (face)	Triangle	Color triplet	SU(3)
Boundary ∂	$\partial^2 = 0$	Gauge closure / anomaly cancellation	—

The ternary complex is not an arbitrary choice. It is the unique simplicial realization of the three-generator SRC set in 3+1 dimensions. The gauge group identification is a theorem of the complex, not a postulate.

7. The Primeness of 137 as Structural Evidence

The fine-structure constant $\alpha \approx 1/137.036$ is a dimensionless number that controls the strength of electromagnetic coupling. Its value is measured but not derived by any current theory. We observe that the

integer 137 is prime.

Within the distinction algebra, a counting invariant is a number that cannot be further factored by the generators of the algebra. A prime number is, by definition, irreducible under integer multiplication. The claim is:

Structural Evidence (not proof): Primeness of 137

If $\alpha_{\blacksquare}^1 \approx 137$ encodes a counting invariant of the distinction algebra over the three-generator SRC set in 3+1 dimensions, then 137 must be prime: a composite value would indicate a factorization into sub-generators that contradicts the minimality of the SRC set.

The observed primeness of 137 is consistent with this requirement. It is not a derivation of α ; it is structural evidence that α is a fundamental counting invariant rather than an accidentally small ratio.

Precision value: $\alpha_{\blacksquare}^1 = 137.035999177(21)$ (CODATA 2022). The nearest prime below and above 137 are 131 and 139, providing no obvious arithmetic structure. 137 itself is prime.

This section makes no claim to derive the numerical value of α . The primeness of 137 is presented as evidence, not proof, consistent with the framework's requirement that its fundamental counting invariants be irreducible.

8. Falsifiability and Experimental Contact

A framework derived from a single axiom is falsifiable if it makes predictions that could be otherwise. The following three predictions follow directly from Sections 2–7:

Prediction 1. No fourth fermion generation. The SRC theorem permits exactly three. Discovery of a fourth generation of quarks or charged leptons would falsify the framework.

Prediction 2. No stable non-Lorentzian signature at any scale. The 3+1 derivation of Result I is not an approximation; it is a logical necessity under Axiom D. Any observation of effectively 2+1, 4+1, or other-dimensional physics at accessible energies would falsify the framework.

Prediction 3. Gauge group cohomological closure. The ternary simplicial complex predicts that any extension of the Standard Model gauge group must preserve the 0-1-2 simplex structure, forbidding certain gauge extensions (e.g., additional U(1) factors that do not arise from boundary operators of the complex). This is testable against beyond-Standard-Model searches at colliders.

9. The Four Open Problems: The Mountains

The framework reaches four boundaries at which it currently cannot proceed without additional structure. These are not failures; they are precisely delimited frontiers that define the next work.

Mountain 1: Fermion masses and mixing angles. The SRC theorem counts generations but does not assign masses. The Yukawa couplings and the CKM/PMNS mixing matrices are not derived. A mechanism within the distinction algebra for symmetry breaking of the three-generation degeneracy is required.

Mountain 2: The numerical value of α . Section 7 identifies 137 as a structural candidate but does not derive 137.036. A first-principles computation of the electromagnetic coupling strength from the ternary complex geometry remains open.

Mountain 3: Quantum-gravitational closure. The framework produces the gauge group at the cohomological level and derives the 3+1 arena, but the dynamics of distinction propagation in curved spacetime—the analogue of quantum gravity—is not developed. The SRC structure must be shown to be compatible with general covariance.

Mountain 4: The origin of Axiom D. The framework takes distinction as given. It does not explain why the universe is a distinction-making structure rather than an undifferentiated whole. This is the deepest open question and likely lies outside the reach of formal physics.

The mountains are named, measured, and left standing. A framework that knows its limits is more trustworthy than one that claims none.

10. Relation to Prior Pillars in the GR-Razor Series

Pillars 1–17 of this series established, through quantitative falsification, that Λ CDM is internally incomplete and that the Cosmological Pangaea framework accounts for cosmic structure, the arrow of time, matter–antimatter asymmetry, small-scale galaxy behavior, and the final state of the universe from a single geometric initial condition.

Pillar 18 operates at a deeper level. Where Pillars 1–17 ask 'given spacetime and the Standard Model, what cosmological initial condition is required?', Pillar 18 asks 'where do spacetime and the Standard Model themselves come from?'. The distinction framework is not a replacement for the Pangaea cosmology; it is the substrate from which the Pangaea arena is drawn.

The Navigable Universe synthesis (archived separately) documents how entropy as fundamental substrate connects the distinction algebra of Pillar 18 to the entropic geometry of Pillars 13–17. The two bodies of work are complementary descriptions at different levels of the same underlying structure.

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